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# Ferroelectric properties of Cd. "Zn. Te solid solutions

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Measurements of the spontaneous polarization P, x-ray diffraction, birefringence, dielectric constant at different frequencies, and specific heat  $C_p$  of the  $Cd_{0.9}Zn_{0.1}Te$  alloy are presented. The results demonstrate that this system exhibits a diffuse, second-order ferroelectric transition.

paraelectric phase; (b) the heat capacity is given by  $C_p = (T_c/C) |(PdP/dT)|$ , where C is the Currie constant. One of the main phenomena observed in these solid solutions is the instability of the formula of the main phenomena observed of the transition terms is needed the

terms of a two state configuration coordinate diagram

## **I. INTRODUCTION**

We will suggest a hypothesis for this type of behavior in terms of a two-state model. Its clarification is a subject for

found to be terroelectric." I his is the first example of fer-	¢
replactricity (ED) in ring blands based avatame. The av	12 元12 金属合体化成合大 医萨里克化 今
permitting to the for such a conclusion was the other for	Single $Cd_{0.9}Zn_{0.1}Te$ crystals were grown by the modi-
ferroelectricity in these solid solutions was given by x-ray diffraction <sup>2,3</sup> and by birefringence measurements. <sup>4</sup> The x-ray-diffraction measurements provide an unambiguous proof of a phase transition through the determination of	We do not use any procedure to polarize the samples. The samples were cut from the grown crystal, polished, and kept in darkness for some days before beginning the mea- surements. When needed, metal contacts were prepared by electroless deposition of gold from a gold chloride solution,

plende structure and that the low-temperature phase is

ferroelectric properties of the  $Cd_{1-x}Zn_xTe$  solid solutions; (ii) about that different executives such as the relative the specific heat, and the birefringence, can be related to one another as observed in other ferroelectrics: (iii) give measurements of the dielectric constant at different frequencies; and (iv) discuss the instability of the ferroelectricity in terms of a configuration coordinate model. To conetched in a 5% Br in methanol solution for about 30 s. I his procedure yielded low-resistance contacts with essentially mean 1, curves, it is the equipment used to near or coor the samples is standard and we shall only give a brief de-

#### UI POLARIZATION VERSUS TEMPERATURE

does not exhibit FE at any temperature, and one that does (below a transition temperature). Indeed, while as-grown samples usually exhibit ferroelectricity, samples heated just integration of the pyroelectric coefficient proposed by Chynoweth<sup>8</sup> to which we have applied a new analysis. The first method can be used at low temperatures, but is of

to reorganization of ferroelectric domains is the determination of the structure by x-ray diffraction: After heating, these samples have a cubic structure at room temperature. a hat sample (uncerton [111]) is manimated by a motalated light from a Ne-He laser beam. The direction of the polarization, as deduced from previous measurements,<sup>1</sup> was  $\langle 111 \rangle$ . The current produced in the samples can have

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FIG. 1. Pyroelectric coefficient  $p_0$  vs temperature T.

three origins: the photoelectric effect, the thermoelectric effect, and the pyroelectric effect. The photoelectric current is eliminated by covering the illuminated gold electrode with graphite. At the same time, we are assured that all the light is absorbed and heats the sample. Appendix A<sup>9,10</sup> describes the method that allows the distinction and separation of the two other contributions: thermoelectric and

roelectric coefficient measured on a sample in which the pyroelectric current is much larger than the thermoelectric current. The pyroelectric coefficient  $p_0$  is deduced from neat capacity obtained independently (see Sec. VII below). Note the neak in  $n_{\rm c}(T)$  at  $T = 105 \,^{\circ}{\rm C}$  Figure 2 shows directly from the hysteresis loop and from integration of the pyroelectric coefficient  $p_0(T)$  (with the reasonable as-



FIG. 2. Temperature dependence of polarization P as obtained from two different techniques: hysteresis loop and integration of the pyroelectric coefficient.

frequencies. Note that at 300 MHz the transition is no longer visible.

sumption that p=0 at T=140 °C; see Fig. 1). There is a good agreement between the two curves when both methods are used at T < 100 °C. The curve deduced from  $p_0$ shows clearly that the transition is diffuse, rather than sharp, in accordance with the measurements of the dielectric constant.<sup>1</sup>

In Ref. 1, measurements of the dielectric constant  $\epsilon$ have been presented at a single frequency of 100 MHz. In the present investigation, the measurements are ex-85U8A vector voltmeter associated with a reflectometer. experimental setup is computerized Figure

at three frequencies (100, 200, and 500 mills) for around the transition temperature. The frequency has a strong effect on the amplitude of  $\epsilon$ : It decreases as the frequency increases. At 300 MHz, the transition is no longer visible. At the same time, the maximum in  $\epsilon(T)$  is shifted to a higher temperature when the frequency increases. These observations are similar to those noted in other ferroelectric solid solutions such as Pb(Zr,Ti)O3 and disordered compounds such as  $Pb(Mg_{1/3}Nb_{2/3})O_3$  and  $Pb(Sc_{1/2}Nb_{1/2})O_3$ .<sup>11</sup>

The absolute value of the slope  $d\epsilon/d\omega$  is maximum for a frequency equal to the inverse of a characteristic time  $\tau$ . In an ideal ferroelectric this time is the relaxation time associated with the polarization dynamics,12 but for a diffuse transition it has been shown<sup>13</sup> that here is a distribution of relaxation times and  $\tau$  is a mean value of the distribution. In our case, a strong decrease of  $\epsilon$  appears between 100 

 $(2\omega \times 10^8)^{-1}$  and  $(6\omega \times 10^8)^{-1}$ , i.e.,  $\tau \sim 10^{-9}$  s. This relatively high relaxation time is indicative of an orderdisorder transition.<sup>12</sup>



FIG. 4. Dielectric constant  $\epsilon$  as a function of temperature T at 100 MHz on heating (O) and cooling (X) the sample. During cooling, one does not observe the anomaly in  $\epsilon$ .



FIG. 5. Powder-x-ray-diffraction line [511] of  $Cd_{0.9}Zn_{0.1}Te$  at 35 °C (a) before heating and (b) after quenching from 160 °C.





FIG. 7. Temperature dependence of  $\delta = (dn/dT)/n$  for the ordinary  $(\delta_0)$  and extraordinary  $(\delta_e)$  refraction indices for the Cd<sub>0.9</sub>Zn<sub>0.1</sub>Te sample.

We still do not control the procedure to our satisfaction, and research on a more reproducible and reversible procedure is continuing.

We next consider the dependence of  $\Delta n$  on polarization. Two cases are possible<sup>16</sup> depending on whether the high-temperature phase is piezoelectric: (i) If the high-

 $\Delta n \sim P^2$ ; here, (dn/dT) can exhibit either a jump (if the transition is sharp) or a smooth variation (if the transition is diffuse); (ii) If the high-temperature (paraelectric) phase is piezoelectric  $\Delta n \sim P$ ; here (dn/dT) exhibits either

mum (if the transition is diffuse).

In our case, the high-temperature phase has the cubic zinc-blende structure (as checked by x-ray diffraction) which is piezeelectric. We find a maximum in (dn(dT)) (Fig. 7), hence, the transition is of second-order type, but

o, we get a straight line for temperature above or C. The roggin for the division from the straight line of lower







FIG. 9. Variation of the specific heat  $C_p$  with temperature T.

#### **VII. SPECIFIC HEAT VERSUS TEMPERATURE**

The specific heat was measured as follows: We took a small  $Cd_{0.9}Zn_{0.1}$ Te sample  $(1.1 \times 1.6 \times 1 \text{ mm}^3)$  and covered its face with graphite. This face was exposed to a  $CO_2$  laser beam. The intensity was varied with time as a step function: I=0 for t<0; I=const for t>0. If the heating is uniform and there are no losses, the temperature increase

( $mc_p$ ), where  $c_p$  is the specific heat at constant pressure. When there are heat losses, the slope decreases with time, but it is still a good approximation to suppose that at t=0 the slope is given by the above expression.  $C_p(T)$  was de-

the problem, taking into account that the heating in the sample is not uniform, shows that the error is about 5%.<sup>14</sup> Hence, the method we used has the advantage of simplicity but it is not very precise.

In 115. 3, the variation of  $O_p$  with the temperature is shown. The behavior is consistent with a diffuse transition

sition would have been either a step (mean-field transition)

# VIII. RELATION BETWEEN SPECIFIC HEAT AND POLARIZATION

In ferroelectrics, the Landau–Devonshire (LD) theory<sup>16</sup> gives a good phenomenological analysis of the experimental observations for above, second order transitions. In

un acco the themaition is diffuse as a statishing word --

previously shown, outside the transition region out results are in accordance with a mean-field second-order phase transition (as given by the LD theory):  $P^2\alpha(T_c-T)$  and the ratio of the slopes  $(d\epsilon^{-1}/dT)$  above and below  $T_c$  is very nearly 2.

From the LD expansion<sup>16</sup> of the free energy

$$G = G_0 + \frac{1}{2}\beta(T - T_c)P^2 + \gamma P^4, \tag{1}$$

it is possible to show that the boot consulty is related to th

potarization by

 $\underline{C_n - C_n} = -T\beta P \frac{dP}{dP}$ 

FIG. 8. Birefringence 
$$n$$
 vs polarization  $P$  curve showing the linearly

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FIG. 10. Excess specific heat  $\Delta C_{\rho}$  (crosses) and |PdP/dT| (circles) vs temperature T mapped onto the same graph, showing the proportionality of the two quantities

It has been noted<sup>17</sup> that the relation (2) should also be valid for diffuse transitions. We shall thus apply Eq. (2) to our case.



FIG. 11. Schematic configuration-coordinate diagram illustrating the proposed two-state system.

frequency dielectric peak (Fig. 5) and a specine-neat peak (Fig. 9) at  $T_c$ , and a splitting of the zine-blende diffraction peak below  $T_c$  (Fig. 5), once the neighborhood of the transition temperature is reached, most of the time, subsequent cooling leads to the disappearance of these effects.

electric behavior can be restored in some samples by a effects are not understood at the EZZ COMPARE und (pro) an ordinary zinc-blende solid solution that is nonferroelecduced from the dielectric measurements for the 10% samtric, while the system in state II could correspond to a ple. The agreement is very good (probably accidental). statically deformed lattice whose lower overall symmetry Thus formula (2), which links the Curie constant, the spe-(see the x-ray data) permits ferroelectricity. The two states cific heat, and the polarization, is verified in this diffuse can be thought of as two distinct minima in a transition. configuration-coordinate diagram. These minima can be IX. DISCUSSION schematically in Fig. 11. This qualitative model can then The measurements presented in this article, i.e., polarbe used to speculate on the nature of the unstable ferro-

attire is about 5  $\mu$ C/cm, this is comparable to the values

The nearest sum cleanly to overcome the barrier  $L_b$ , part

solutions behave as regular ferroelectrics. However, it is necessary to explain why a small amount of Zn gives such a high transition temperature. Only 4% of zinc gives  $T_c \simeq 90$  °C (363 K). This can be compared with an analogous system  $T_{c} = 16\%$  corresponds only to  $T_c \simeq 220$  K.<sup>18</sup>

A central hallmark of the ferroelectricity in this system is its dependence on the thermal history. For example, while  $\frac{1}{2} = \frac{1}{2} = \frac{1}{$  the ferroelectricity. These particles can be released only if the sample is annealed at a temperature  $T \gg T_c$  to overcome the larger barrier  $E_b^{(2)}$ .

To search for a microscopic model that could result in such a situation, we have studied theoreticality<sup>19</sup> off-tenter atomic displacements in ordinary zinc-blende semiconductors. A "supercell," consisting of four cations and four anions, originally placed at the zinc-blende atomic posi-

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$\begin{array}{c} & & & \\ & & & \\ & & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ \end{array}$	Sample Laser illumination
a) 0 3.0 ↓ 2.5 (b)	FIG. 13. Schema of the pyroelectric and thermoelectric measurements.
	sion studies show that CuCI has $a$ bands hear the top of its valence band, <sup>23</sup> II-VI compounds have considerably deeper
0 0.05 0.1 0.15 0.2	strong anharmonicities, but no distinct secondary mini-
FIG. 12. Results of first-principles LAPW total energy calculations illus- trating the off-center behavior in some zinc-blende semiconductors.	akin to the disordered alloy environment could lead to a metastable minimum. Hence, while the conceptual model envisioned in Fig. 11 seems applicable to $I_B$ -VII systems, direct evidence for its applicability to II-VI systems is lack-
rection) resulting in a C summetry (see the x ray stude	in their vibrational properties (as was seen by Livescu and
Fig. 12. We see that for a system with active cation $a$ orbitals such as CuCl, there exists indeed a secondary, off- center minimum, as envisioned in Fig. 11. The details of the missioned in Fig. 12.	bands, polarization, orennigence, etc. Furthermore, if the dipoles in this system are induced by the Cd-Zn size dif- ference which leads to off-center displacements, a similar effect should be seen in other size-mismatched allows e g
bards, and the construction hands (the "neorda	
As pointed out by Bersuker <i>et al.</i> <sup>22</sup> the <i>t</i> <sub>3</sub> -type normal- mode displacements in $A I_4$ tetranedral systems induce a dipole. In state II there are four local-equilibrium config- urations corresponding to the four bond directions. In each minima, the dipole coincides with the direction of the	ACKNOWLEDGMENTS The technical help of A. Kaanan and F. Kon is ac- knowledged. This research was funded in part by Grant No. 89-00324 from the United States-Israel Binational Sci- ence Foundation (BSF), Jerusalem, Israel, and by the Center for A hearming in Science, Ministry of Immigrated
state II) the system jumps dynamically between these min- ima through tunneling. The observed net dipole will then depend on temperature and tunneling rate. In the absence of correlation between the tetrahedra, the system as a whole will be unpolarized. As the temperature is lowered,	Absorption, State of Israel.          APPENDIX A         The experimental situation for the measurement of the

The experimental situation for the measurement of the pyroelectric current is shown in Fig. 13. A slab (thickness d) is illuminated by a modulated laser beam [intensity  $W_1$  +  $W_0 \exp(j\omega t)$ ]. The thermal contact of the second face with the holder is characterized by a relaxation time  $\tau$ . If other dimensions are larger than the thickness, the excess temperature will depend only on x and t. In the steady-state regime<sup>9</sup> T is equal to

electric zinc-blende structure.

the dipole correlation is enhanced and a net polarization

will appear as a result of an order-disorder transition. Note

that the ferroelectricity in such a system is metastable fer-

roelectricity, since the true ground state has the nonferro

Since the off-center displacement found in CuCl derives from a metal *d-s* coupling, it is expected to be weaker

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FIG. 14. Current l vs log f for a CdTe sample. The line is a fit following Eq. (A3).

FIG. 15. Current I vs log f for a Cd<sub>0.9</sub>Zn<sub>0.1</sub>Te sample, showing that for large frequencies the current goes toward a finite value. The line is a fit obtained using expressions (A3) and (A5).

$$\Delta T(x,t) = \frac{c}{C_p \rho d} \left[ \frac{1}{1 + \gamma \sigma \tau_1} + 2 \sum_{n=1}^{\infty} \cos\left(\frac{\pi d}{d}\right) \right]$$
total current is the sum of the two contributions.  
The absolute value  $f_{ab}$  is characterized by a monotonic decrease when  $\phi$  increases from zero to  $\phi$ . There is a sample of that was of interest. C is the heat canacity  $\phi$  is the sample with  $\gamma$  is a sample with  $\alpha$  by protectric energy and this means that the protectric contribution if  $f_{ab} = (K/R) [\Delta T(x=0) - \Delta T(x=d)],$  (A2)  
where  $K$  is the themoelectric power and  $R$  is the total residue  $f_{ab}$  is a solor or much larger than  $f_{ab}$  is no frequency and this means that the protectric contributions is a solor or much larger than  $f_{ab}$  has no frequency and this means that the protectric contribution if it was alone or much larger than  $f_{ab}$  has no frequency dependence. As shown in Fig. 15 by the solid line, the fit is very good.  
To verify the validity of this expression a sample of QAT ( $x = d$ ).  
 $f_{ab} = \frac{C}{d} \int_{0}^{d} \frac{d\Delta T}{dt} (x,t) dx$ , (A3)  
summe of the two contributions is similar to the sample use of  $f$  on  $\phi$  is similar to the contribution is  $f_{ab} = C_p f d + \gamma \phi \tau_1$  (A3)  
summa protection of the fit and all the other known quantities is  $2O_0V/K$ . It compares well with the results of Kubalkova in  $G^{T} = C_p f d + \gamma \phi \tau_1$  (A3)  
summa is the protectric operation of  $f$  on  $\phi$  is similar to  $f^{T} = C_p f d + \gamma \phi \tau_1$  ( $f^{T} = 0$ )  
 $f^{T} = C_p f d + \gamma \phi \tau_1$  ( $f^{T} = 0$ )  
 $f^{T} = C_p f d + \gamma \phi \tau_1$  ( $f^{T} = 0$ )  
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 $f^{T} = C_p f d + \gamma \phi \tau_1$  ( $f^{T} = 0$ )  
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$$AP_{x} + BP_{x}^{3} + CP_{x}(P_{y}^{2} + P_{z}^{2}) = 0,$$

$$\frac{2}{2H_{y} + DT_{y} + CT_{y}(T_{x} + T_{z}^{2}) = 0,$$

$$AP_{z} + BP_{z}^{3} + CP_{z}(P_{x}^{2} + P_{y}^{2}) = 0.$$
(B2)

The rhombohedral solution is  $P_x = P_y = P_z = P_0$  with  $P_0$  given by

$$A + (B + 2C)P_0^2 = 0.$$
 (B3)

From Eq. (B3), one sees that  $P_0$  varies as  $(T_c - T)^{1/2}$ .

Now we apply electric field  $(E_x, E_y, E_z)$ . From  $dG/dP_x + E_x$  (and similar expressions for  $P_y$  and  $P_z$ ), one finds (for  $T < T_c$ )

$$E_{x} = 2BP_{0}^{2}\delta P_{x} + 2CP_{0}^{2}\delta P_{y} + 2CP_{0}^{2}\delta P_{z},$$

$$E_{y} = 2BP_{0}^{2}\delta P_{y} + 2CP_{0}^{2}\delta P_{x} + 2CP_{0}^{2}\delta P_{z},$$

$$E_{z} = 2BP_{0}^{2}\delta P_{z} + 2CP_{0}^{2}\delta P_{x} + 2CP_{0}^{2}\delta P_{y},$$
(B4)

$$P_x=P_0+\delta P_x, P_y=P_0+\delta P_y, P_z=P_0+\delta P_z.$$

The components of the susceptibility tensor  $\chi_{ij}$  are  $\delta P_i/E_j$ . From Eq. (B4) it is easy to see that all the  $\chi_{i,j}$ 's are pro-

### the polarization.

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is. In our case, it means that an anomaly must always be

The slope  $d\epsilon^{-1}/dT$  is dependent on the chosen direc-

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