(1985) with the important distinction that we use the two-way propagators in our formulation of the problem and suppress only evanescent waves. We provide examples and discuss some implications of our approach later in the paper.

We observe that to suppress only the evanescent waves, it is necessary to use a spectral projector on a subspace spanned by eigenfunctions corresponding to the propagating modes. In fact, in the case of depth-dependent background, a composition of the Fourier transform and the ideal cutoff filter in Kosloff and Baysal (1983) constitutes such a projector.

For a general variable background, by using spectral projectors we arrive at a well-conditioned initial-value problem in the spatial variable. We note that an algorithm for computing spectral projectors does not necessarily involve computing the eigensystem for the propagator, most often a prohibitive proposition. The spectral projector (simply related to the sign function of a matrix) may be com-

$(\mathbf{x_s},\mathbf{x_r}, \ ,\square)$ to $_{+1} = \ + \square$. We then take the resulting survey and extrapolate in depth all receiver gathers, again by a step $\square$. Fi-

(d) Update the migrated image $I(\mathbf{x}, \ _{+1}) = I(\mathbf{x}, \ _{+1}) + \ (\mathbf{x_s} = \mathbf{x}, \mathbf{x_r} = \mathbf{x}, \ _{+1}, \square)$

For propagating the wavefield from to $_{+1}$, we use an algorithm based on Coult et al. (2006) that has practically insignificant numerical dispersion. We note that in some cases the source-receiver configuration may allow the same spectral projector to be used for downward continuing several gathers. For example, if the source and receiver grids coincide, so do the corresponding projectors.

We also note that if the velocity varies only in depth but not horizontally, then applying the spectral projector to the operator is equivalent to applying an ideal low-pass filter in the spatial wavenumber domain. In such situations, our method is similar to the method proposed in Kosloff and Baysal (1983), as noted earlier in the discussion of ill-posedness of downward continuation.

We implement the absorbing boundary conditions using a variant of the approach of Cerjan et al. (1985). This allows us to decouple the application of the absorbing boundary conditions from the application of spectral projectors. As a result, we need to compute the spectral projector in a slightly extended domain but only for either periodic or zero boundary condition. As a result, the operator on the extended domain remains self-adjoint so that we may use the algorithm in Appendix A.

Computing costs

We estimate the computing costs of 3D survey sinking except for the cost of computing spectral projectors which we estimate only in two dimensions. Computing spectral projectors in three dimensions is part of further research, and we expect significant savings by developing fast algorithms for this purpose. Let and

Generating the survey

We generate data for our experiments using slice 337 in the crossline dimension of the SEG-EAGE model (Figure 2a). The same slice was used by Stoffa et al. (2006). The input model has the physical dimension 13,500 □ 4000 m.

The receiver data was generated by using the modeling algorithm described by Coult et al. (2006) with absorbing boundary conditions at the sides. We used a Ricker pulse with a dominating frequency of 7 Hz, and recorded a 12 s time trace for each shot. We placed sources at shot locations = □ , = 0, …, 675 where □ = 20 m. For each source , we placed receivers at $_{,}$ = + □ , = − 68, …, 68. Hence, the receiver aperture for each shot corresponds to 2700 m, or one-fifth of the lateral extent of the domain, except for sources near the boundaries where the receiver array was truncated in order to fall within the modeling domain.

Example 1

As background velocity for the migration algorithm, we used a blpo3reasitag0.333onr(ag0.3e0TD)T-0.63780.9995TD()Tj/F81Tkam)Tjr-181.5ovthscribed in the Algorithm section. The result is shown in Figure 4b.

Comparing Figure 4a and b, we note that the shape of the salt body is remarkably well preserved despite the inexact velocity model. We also note that although the velocity model in Figure 2c does not contain significant velocity variations, the spectral projector method still gives a significant advantage over the method using an ideal low-pass filter.

CONCLUSION

Migration schemes based on factorization of operator L in equation 3 into up- and downgoing waves produce errors because of suppression of propagating waves and, in a variable background, because of approximate factorization of the operator. Alternative approaches in variable background that exist today are two-way equation schemes based on using the initial-value problem in time. Such reverse-time migration schemes change the inverse problem so that "local interactions" between events are now in time rather than in depth. A careful comparison of our full-wave-equation depth extrapolation for migration with that of reverse-time migration is beyond the topic of this paper and should be a subject of further research.

Our formulation of the downward-continuation operator removes only nonpropagating evanescent waves, thus preserving propagating waves moving in all directions. We have demonstrated significant improvement in imaging by comparing our approach to that of a method where most but not all propagating waves are preserved, hence emphasizing the sensitivity of imaging to the erroneous removal of propagating waves.

While our method is computationally more expensive than some simpler techniques, the quality of the results justifies the effort to develop fast 3D algorithms for this type of migration and inversion. We plan to develop our approach further to a full 3D version and work on making our algorithm competitive with other migration methods in terms of speed. We also plan to test full-wave-equation depth extrapolation on real data. Looking beyond these remaining issues, the results of this paper indicate many new interesting possibilities to advance seismic methods, such as to include multiple reflections into image formation and to improve the velocity analysis.

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APPENDIX A

COMPUTING SPECTRAL PROJECTORS

To compute the spectral projector on the negative part of the spectrum in equation 9, we use a simple iteration scheme (see e.g., Kenney and Laub, 1995; Auslander and Tsao, 1992; and Beylkin et al., 1999).

For a self-adjoint matrix L, the spectral projector $\mathcal{P}$ is simply related to the sign function of a matrix, namely $\mathcal{P} = (I - \text{sign}(L))/2$. In order to find $\text{sign}(L)$, we iterate according to

1. Initialize $\mathcal{S}_0 = L/\|L\|_2$.
2. **For** $\ = 1, \ldots, \ $:

$$\mathcal{S}_{\ +1} = \frac{3}{2}\mathcal{S}_{\ } - \frac{1}{2}\mathcal{S}_{\ }^3.$$

The iteration converges quadratically, $\mathcal{S}_{\ } \rightarrow \text{sign}(L)$. For details on analysis of this iteration, see Beylkin et al. (1999), although the basic proof is simple. Noting that all matrices $\mathcal{S}_{\ }$ are diagonalizable by the same transform, this iteration needs to be verified only in the

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