

On the finiteness in the deformed Hamiltonian mean-field model

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Abstract: We study the finiteness of the number of islands in the deformed Hamiltonian mean-field model. The model is defined by a Hamiltonian with a periodic potential and a linear perturbation. The number of islands is shown to be finite for a large class of parameters. The proof is based on the theory of normal forms and the KAM theorem. The number of islands is bounded by a function of the perturbation strength and the frequency ratio. The results are illustrated by numerical simulations.

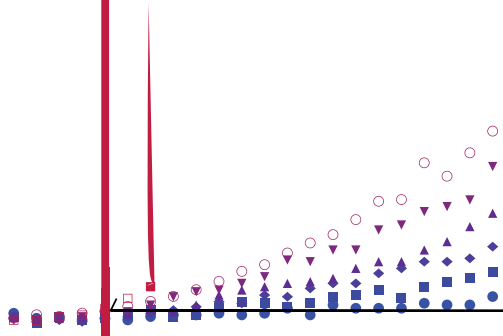
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I. INTRODUCTION

The Hamiltonian mean-field model (HMF) is a paradigmatic model of a many-body system. It consists of particles interacting via a long-range Coulomb force. The model is integrable in the limit of zero temperature and zero interaction strength. In the present paper, we study the finiteness of the number of islands in the deformed HMF model. The deformed HMF model is defined by a Hamiltonian with a periodic potential and a linear perturbation. The number of islands is shown to be finite for a large class of parameters. The proof is based on the theory of normal forms and the KAM theorem. The number of islands is bounded by a function of the perturbation strength and the frequency ratio. The results are illustrated by numerical simulations.

The evolution of the density is given by the continuity equation



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