of optical tunneling whereby a dark soliton incident upon a spatially extended hydrodynamic barrier in the form of a DSW or a RW can penetrate through to the other side of the evolving hydrodynamic structure. Thus, in contrast to the traditional notion of soliton tunneling through an externally imposed barrier, hydrodynamic soliton tunneling corresponds to the full penetration and emergence of the soliton through an intrinsic hydrodynamic state that evolves according to the same equation as the soliton. This generalizes the understanding of a soliton as a coherent, particle-like entity that can interact elastically with other solitons [28] and dispersive radiation [29] to one that can also interact with nonlinear hydrodynamic states and emerge intact, i.e., without fissioning or radiation, albeit with a different amplitude that results from a change in the background mean flow.

In this paper, we analyze the tunneling of solitons through hydrodynamic states within the framework of the integrable, defocusing nonlinear Schrödinger (NLS) equation, which is an accurate model for nonlinear light propagation in single mode optical fibers with normal dispersion [30]. We invoke the scale separation L inherent to Whitham modulation theory in order to derive a system of asymptotic equations that describe the interaction between narrow dark solitons and evolving, broad hydrodynamic barriers. We obtain the conditions on the incident soliton amplitude and hydrodynamic mean flow density and velocity for tunneling. One of the fundamental properties of hydrodynamic soliton tunneling is hydrodynamic reciprocity whereby the tunneling through RWs and DSWs is described by the same set of conditions in spite of the very different interaction dynamics. This general property of solitonic hydrodynamics has been recently formulated and experimentally confirmed for a fluid system [31]. We also show that tunneling is not always possible and that the soliton can be absorbed or trapped within the hydrodynamic flow. Moreover, we find that soliton interaction with hydrodynamic states can lead to reversal of the soliton's propagation direction and spontaneous soliton cavitation.

Our analysis can be applied to a large class of dispersive hydrodynamic systems, including dispersive Eulerian equations [23,32] which have broad applications. The particular case of optical hydrodynamic soliton tunneling considered here could be observed, for example, within the experimental setting described in Ref. [22] for the generation of DSWs and RWs in optical fibers. This work generalizes unidirectional solitonic hydrodynamics to the optical setting where waves can propagate bidirectionally.

II. PROBLEM FORMULATION

We consider the defocusing NLS equation

$$i\psi_t = -\frac{1}{2}\psi_{xx} + |\psi|^2\psi, \qquad (1)$$

where in the context of fiber optic propagation, t is the longitudinal coordinate in the fiber, x is the retarded time, and $\psi(x,t)$ is the complex-valued, slowly varying envelope of the electric field. All variables are nondimensionalized to their typical values. See, e.g., Ref. [22] for a detailed description of NLS normalizations and typical values of physical parameters pertinent to the regimes considered here.

Equation (1) can be written in dispersive hydrodynamic form via the transformation $\psi = \sqrt{\rho}e^{i\phi}$, $u = \phi_x$:

$$\rho_t + (\rho u)_x = 0, \quad u_t + uu_x + \rho_x = \left(\frac{\rho_{xx}}{4\rho} - \frac{\rho_x^2}{8\rho^2}\right)_x, \qquad (2)$$

where ρ is the optical power and u is the chirp. In terms of the hydrodynamic interpretation of these quantities, we will refer to ρ as a mass density and u as a flow velocity (see, e.g., Ref. [23]). Within this setting, the normalized coherence length is $\ell = \rho_0^{-1/2}$ where ρ_0 is a typical density scale. The coherence length is an intrinsic scale that, along with the coherence time $\tau = \rho_0^{-1}$, corresponds to a scaling invariance of the hydrodynamic equations (2). (In BECs, ℓ is

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$$\bar{\rho} - a\,\mathrm{sech}^2[\quad)],$$

$$\bar{u} \quad 1 \quad \sqrt{\quad} - a[- \quad (x,t)],$$

$$c = \bar{u} \quad \sqrt{\quad - a}, \qquad (3)$$

relation. The ± in (3) due to the bidirectional

NLS escrr4/F6a a dispersive hydrodynamic sys

is a zero density, cavitation point.

The [ypicaltunneling [problem consists of a [solito

on a fixed potential barrier, either due to a change in the

or an external effect. However, the spatio-tempora

considered here evolve according to the same eso

describes the dynamics of the medium. For an op

with homogeneous, normal dispersion, this corresp

time-dependent input signal that results in boioL T9

tions[for thew

$\bar{\rho}$, u $\bar{u}$:

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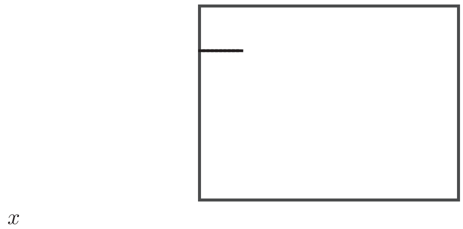
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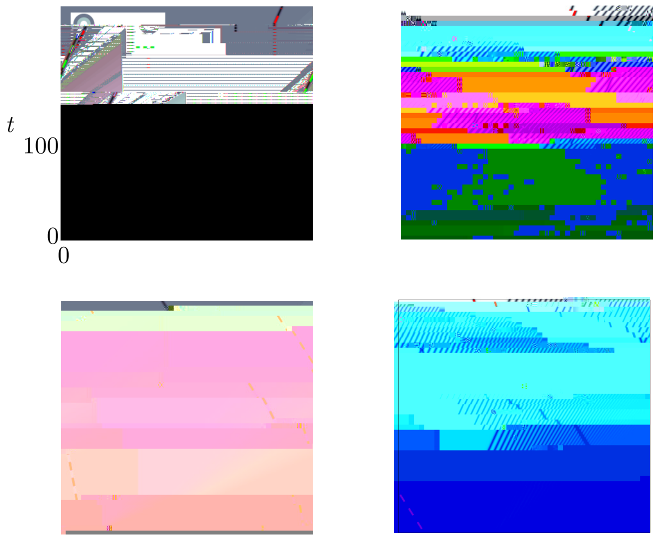
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determining the resulting amplitude, velocity and shift of the solitary wave post-interaction. The methodology presented here to track the trajectory of the soliton only requires knowledge of the far field boundary conditions and hence this approach can be extended to other initial configurations. We also note that the developed theory is not restricted to integrable NLS dynamics and can be generalized to other cases of hydrodynamic optical soliton tunneling described by nonintegrable versions of the defocusing NLS equation, e.g., with saturable nonlinearity, using the methods of Refs. [23,32,44].

ACKNOWLEDGMENTS