Fast algorithms for Helmholtz Green's functions

B G. EG . BE I, *, CH. HE. . C. A, D CA , ,

2 , , 030 -0 2 ,

H G '

G ,

G ,

H. G , ,

= K ,

B. E (1921)
(1.6) (G & (1980) (1998)

Proposition 2.1. ([illegible]) [illegible] $\in \mathcal{S}(\mathbb{R}\ )$, Λ [illegible]

[illegible], Λ^* [illegible]

3. Quasi-periodic Green's function via absolutely convergent series

3... - . ' . G .. ' .. ' .. ' (1.6) . .

( ), (1.1) (1.4) (1.5).

Proposition 3.1. (3.1) (1.2) (1.3) > 0, $\neq 2$ $\boldsymbol{d}$ K , $\boldsymbol{d} \in \Lambda^*$ $\in \mathbb{R}$ ≥ 2.

(3.1)

F C $= \frac{1}{\;}$ $\sum_{\boldsymbol{d}\in\Lambda^*}$ $\frac{\left(\frac{K\,2\;\boldsymbol{d}K\;^2 C\;^2}{4\;^2}\right)}{2\;\boldsymbol{d}\,K\;^2\,K\;^2}$ \$ 2 $\boldsymbol{d}$K K \$ $=$ K \$ F ,

3 3 1

2 \$$\boldsymbol{d} = 1$ $\in \Lambda$ $\boldsymbol{d}$

$$= \frac{1}{2}$$

Remark 3.3. [illegible] G [illegible]

I (3.12), 2.1

$>$.

$$\frac{1}{2^{3/2}} \quad 0 \quad \frac{C \quad ^2}{4^2} \quad \in \Lambda \quad \$ \quad K \quad C \; ^2 \; ^2$$

$$= \frac{1}{2} \quad 0 \quad \frac{C \quad ^2}{4^2} \quad \sum_{\boldsymbol{d} \in \Lambda^*} \$ \; 2 \; \boldsymbol{d} K \quad K \frac{2 \; \boldsymbol{d} K \quad ^2}{4^2} \quad \frac{}{3},$$

Λ^* . B

,

$$\frac{1}{2} \sum_{\boldsymbol{d} \in \Lambda^*} \$ \; 2 \; \boldsymbol{d} K \quad 0 \quad \frac{K \; 2 \; \boldsymbol{d} K \; ^2 \; C \quad C \; ^2}{4^2} \quad \frac{}{3}$$

$$= \frac{1}{} \sum_{\boldsymbol{d} \in \Lambda^*} \frac{\$ \; 2 \; \boldsymbol{d} K \quad \frac{K \; 2 \; \boldsymbol{d} K \; ^2}{}}{}$$

E T 2 4 7 . 5 . 5 6 0 . 5 4 6 9 0 j/ F 2 1 1 7 . 6 7 1 1

4. Fast convolutions with Green’s function

G ’ (3.1) (3.2) (3.2) G G ’ , ( ) ,

( ) ’

G ’ (3.4).

F ,
F

$$\tilde{}_{F} = \frac{1}{} \sum_{\substack{d\in\Lambda^* \\ 2 \; d K \le}} \frac{\left(\frac{K\; 2 \;\; dK \;^2 C \;^2}{4 \;^2}\right)}{2 \;\; d K \;^2 K \;^2} \; \$ \; 2 \;\; dK , \qquad 4.1$$

> 0 > 0

F
(3.3)
G , F , > 0
(3.2)

$$\sum_{\substack{\in\Lambda \\ \lesssim}} \$ \; C ,$$

, , B. ( ) G ,

$$= \sum_{=1} K \;^2 , \qquad 4.2$$

> 0 > 0.
(B. ( )). (4.2),

$$\tilde{} = \sum_{\substack{\in\Lambda \\ \lesssim}} \$ \; C . \qquad 4.3$$

C (4.1) (4.3), G ’

$$\tilde{} = \tilde{} \quad C \tilde{}_{F} . \qquad 4.4$$

: ( )

( )

(4.2).

$\tilde{}_{F}$ F

$$\tilde{}_{F} * = \frac{1}{} \sum_{\substack{d \in \Lambda^* \\ 2 \; d K \leq}} \frac{\left(\frac{K\, 2 \;\; d K \;{}^2 C \;{}^2}{4 \;{}^2}\right)}{} \qquad d \; d K \quad 2\,2 \quad d \quad K$$

> 0 > 1

$$\frac{1}{} \sum_{\substack{\boldsymbol{d}\in\Lambda^* \\ 2\ \boldsymbol{d}\mathrm{K} >}} \frac{\left(\frac{\mathrm{K}\,2\ \boldsymbol{d}\mathrm{K}\ ^2\mathrm{C}\ ^2}{4\ ^2}\right)}{2\ \boldsymbol{d}\mathrm{K}\ ^2\mathrm{K}\ ^2} \leq \frac{}{3}$$

,   ,

$$\left\| \ \mathrm{F} \quad \mathrm{K}\,\tilde{\ }\,\mathrm{F} \ \right\|_{1} \leq \frac{}{3}. \qquad 4.8$$

$\mathrm{K}\,\tilde{\ }$

$| \quad |^{1}$

(3.1) ≥

$$\frac{\left(\frac{K\;\;{}^{2}K^{2}}{4^{2}}\right)}{{}^{2}K^{2}} \le \frac{}{{}^{2}\;{}^{2}K\,1}.$$

(4.16), (3.2) A > 1, F ∼3,

Remark 4.2. D E ’ (C (1978) (1986) = 0). (2006; 2006 B). F

( ) F , B. (2008), F $\mathcal{O}($ $)$, $\sim$ A , $\sim$ C 2 K1 .

A :

( ) B. (2008), (4.7) $\mathcal{O}$ \$ \$ \$. A G (G & 1991; 1991; G & 1998) , . A \$ \$ $\sim$ K1 3, (, , B. (2008) G). (4.7) $=0$ (4.7) $\mathcal{O}$ C 3 K1), 3 .

( ) F F 2 dK $\leq$ K1 (3.1). G (4.5), FF (D & 1993; B. 1995; & G 2005) $\mathcal{O}$ C C $\mathcal{O}$, $\mathcal{O}$ C 4 K1 , 4 .

F ,

(4.17) G ,

. B

G ,

. F

, G ,

(4.17) (4.17) ,

$= \Delta$ C (2000, (17)) Δ

(1998, (2.49), (2.53) (2.54))

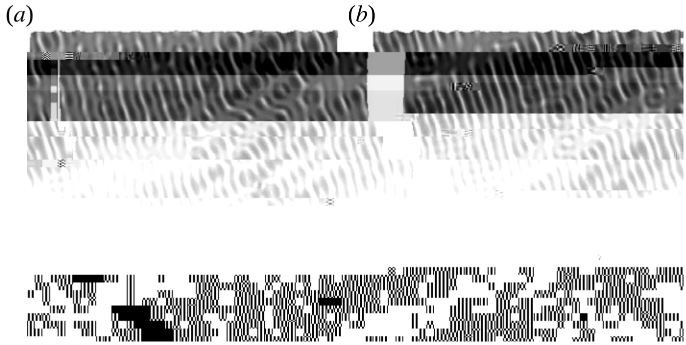
I 1, (4.17)

(4.4) $\approx 10^{K9}$. $= 0$

($\approx 10^{K9}$) 4.1.

$$= \frac{2\alpha}{\ } \ K \ \ ^{3} \ K\alpha \ K \ C^{2} \quad \in \Lambda \ =1 \tag{4.18}$$

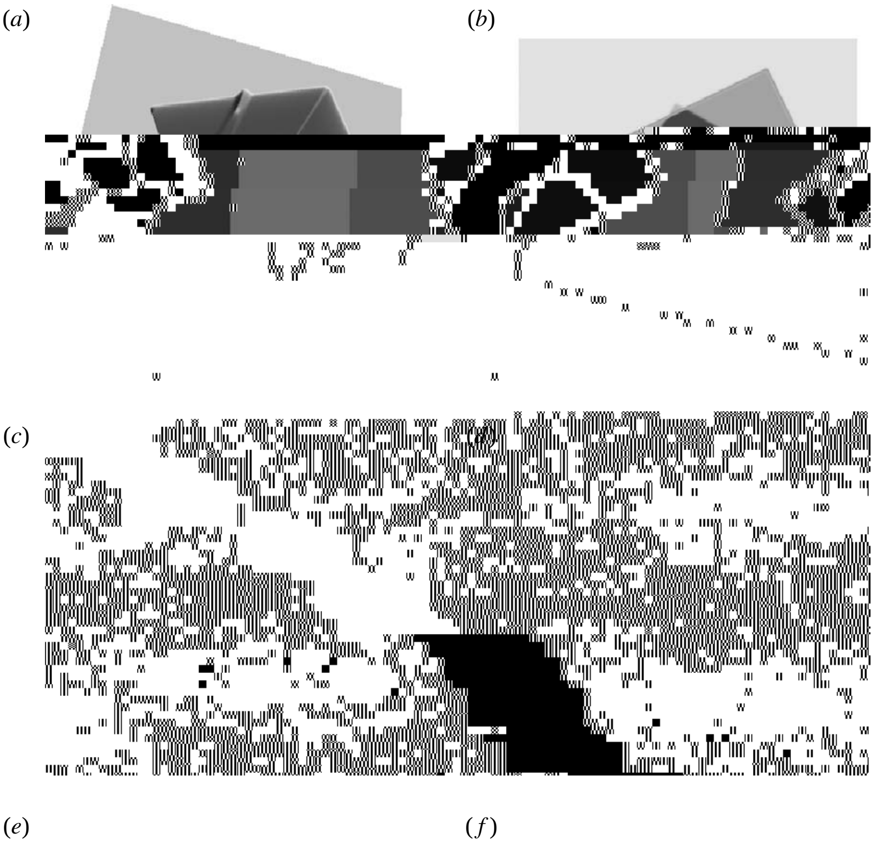
$\alpha = 300$, $= (1/3, 4/7)$, $_1 = (0, 0)$, $_2 = (1/10, 1/10)$ $_3 = (K3/$



F . 3. A - G ' =(3, 5) | =100 [illegible] $_1$= 1, 0 $_2$= 1/2, 3/2 [illegible] K1/2, 1/2 ! K1/2, 1/2 : ( ) ( ) [illegible]

[illegible] F [illegible]. I 2, [illegible] G ' [illegible] =10^{K11}, [illegible] 2 [illegible] 2 ≈ 1.76 [illegible] 2 ≈ 1.31$10^3. [illegible] 4.1. [illegible]

[illegible] G ' [illegible]. I 3, [illegible] G ' [illegible] : ( ) [illegible] F [illegible] ( ) [illegible] G ' [illegible]. I 4, [illegible] G ' [illegible] ( ) [illegible].



5. Green's functions with boundary conditions on simple domains

[illegible] G [illegible] (3.4). [illegible] G [illegible] G [illegible] G [illegible] D [illegible]

F G .

I , G

F

F , D

$= K1/2, 1/2 \; ! \; K1/2, 1/2$.

G , G (2

),

$$\Delta C 4 \; ^2 \quad = K$$

(1.3) $=0$. G

$$_1, \, _2 = K \frac{1}{4} \sum_{_1=K\infty}^{\infty} \sum_{_2=K\infty}^{\infty} \; _0 \; 2 \quad _1 C \; _1{}^2 C \; _2 C \; _2{}^2 ,$$

G , (1.6),

G , $_1 = 1, 0$ $_2 = 0, 1$.

(3.4),

$$_1, \, _2 = \frac{1}{2} \sum_{\in \mathbb{Z}^2}^{\infty} \; 2 \quad K \quad C \; ^2 \frac{2}{4} C \; 4 \; ^2 \; ^{K2}$$

$$C \sum_{\in \mathbb{Z}^2} \frac{\frac{K \; ^2 \; ^2 C \; ^2}{}}{}$$

$$= \left(K\frac{\cdot}{4} \quad K \quad C\,2 \quad ^2\right) K \quad \left(K\frac{\cdot}{4} \quad C \quad C\,1\,C\,2 \quad ^2\right). \qquad 5.3$$

(5.2)

$$\tilde{D} \quad = \quad \sum_{=1} \quad \int_{{}_1^2 C\ {}_2^2 \leq} \quad {}^{1/2}_{K1/2} \quad 2 \quad 2$$

$$! \quad {}^{1/2}_{K1/2} \quad 1 \quad 1 \quad 1 \quad 2 \quad 1 \quad 2,$$

§4.

I F , , >1

(4.8)

$$\tilde{D}_F \quad 1, \; 2, \; 1, \; 2 \;= \int_{2\ {}_1^2 C\ {}_2^2 \leq} \frac{\frac{K\ ^2\ {}_1^2 C\ {}_2^2\ C\ ^2}{2}}{4\ ^2\ {}_1^2\ C\ {}_2^2\ K\ ^2} \quad 1\ 1\ C\ 2\ 2$$

$$! \left({}^{K}\ 1\ 1\ K\ 1\ 1\ C1 \right)\left({}^{K}\ 2\ 2\ K\ 2\ 2\ C1 \right). \qquad 5.4$$

$$\tilde{D}_F \quad = \int_{2\ {}_1^2 C\ {}_2^2 \leq} \frac{\frac{K\ ^2\ {}_1^2 C\ {}_2^2\ C\ ^2}{2}}{4\ ^2\ {}_1^2\ C\ {}_2^2\ K\ ^2} \quad 1\ 1\ C\ 2\ 2$$

$$! \quad 1, \quad 2 \quad K \quad 1 \quad K \quad 1, \quad 2$$

$$K \quad 2 \quad 1, K \quad 2 \quad C \quad 1\,C \quad 2 \quad K \quad 1, K \quad 2 \; , \qquad 5.5$$

(4.6). FF (5.5) §4 .

Remark 5.1. A

Remark 5.2. G D $=3$. I

A ( )

H 2003, 2004; 2004).

I

G

( ) = 0.

H

A G

( ) ( )

( ), ( )

F E

F G

G

F D -0612358, D E/ 4000038129, D E DE-FG02-03E 25583 AF FA9550-07-1-0135.

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