1. (30 pts) Consider the two planes described by $2y + z = 1$ and $\quad x + 2y + 2z = 3$.

   (a) Is the point $P(\quad 1; 0; 1)$ on both planes, one of the planes, or neither?

   (b) Find the angle formed by the two planes.

   (c) Find equations for the line of intersection of these two planes. Express your answer in parametric and symmetric forms.

   (d) What is the shortest distance between the point $Q(0; 1; 3)$ and the line of intersection?

   Solution:

   (a) The point $P(\quad 1; 0; 1)$ satisfies both equations, so it lies on both planes.

   $$0 + 1 = 1 \quad \text{and} \quad 1 + 0 + 2 = 3$$

   (b) The angle $\quad$ between the planes equals the angle between their normal vectors $\mathbf{n_1} = \langle 0; 2; 1 \rangle$ and $\mathbf{n_2} =$

thus

$$d = \frac{|\mathbf{PQ} \times \mathbf{v}|}{|\mathbf{v}|} = \frac{\sqrt{29}}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{\sqrt{29}}{3}.$$

Alternate Solution

The shortest distance equals

$$\begin{aligned}
|\mathbf{PQ} - \text{proj}_{\mathbf{v}}\mathbf{PQ}| &= \left|\mathbf{PQ} - \frac{\mathbf{PQ}\cdot\mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}\right| \\
&= \left|\langle 1, 1, 2\rangle - \frac{\langle 1, 1, 2\rangle \cdot \langle 2, -1, 2\rangle}{3^2}\langle 2, -1, 2\rangle\right| \\
&= \left|\langle 1, 1, 2\rangle - \tfrac{5}{9}\langle 2, -1, 2\rangle\right| \\
&= \left|\langle -\tfrac{1}{9}, \tfrac{14}{9}, \tfrac{8}{9}\rangle\right| = \frac{\sqrt{29}}{3}.
\end{aligned}$$

2. (16 pts) A particle is moving in the direction $\mathbf{v} = \mathbf{i} + \mathbf{j}$ when a force of $\mathbf{F} = 3\mathbf{j} + 4\mathbf{k}$ is applied to it.

   (a) Decompose the vector $\mathbf{F}$ into a sum of 2 vectors: one vector parallel to the particle's direction of motion and the other vector orthogonal.

   (b) Find a unit vector that is orthogonal to $\mathbf{F}$.

Solution:

(a) A vector parallel to the particle's direction of motion is the projection of $\mathbf{F}$ onto $\mathbf{v}$.

$$\text{proj}_{\mathbf{v}}\mathbf{F} = \frac{\mathbf{v}\cdot\mathbf{F}}{|\mathbf{v}|^2}\mathbf{v} = \frac{\langle 1, 1, 0\rangle \cdot \langle 0, 3, 4\rangle}{\sqrt{2}^{\,2}}\langle 1, 1, 0\rangle = \tfrac{3}{2}\langle 1, 1, 0\rangle = \left\langle \tfrac{3}{2}, \tfrac{3}{2}, 0 \right\rangle.$$

A vector orthogonal to the projection is

$$\text{orth}_{\mathbf{v}}\mathbf{F} = \mathbf{F} - \text{proj}_{\mathbf{v}}\mathbf{F} = \langle 0, 3, 4\rangle - \left\langle \tfrac{3}{2}, \tfrac{3}{2}, 0 \right\rangle = \left\langle -\tfrac{3}{2}, \tfrac{3}{2}, 4 \right\rangle.$$

Therefore

$$\mathbf{F} = \langle 0, 3, 4\rangle = \left\langle \tfrac{3}{2}, \tfrac{3}{2}, 0 \right\rangle + \left\langle -\tfrac{3}{2}, \tfrac{3}{2}, 4 \right\rangle.$$

(b) The vector $\mathbf{F} = 3\mathbf{j} + 4\mathbf{k}$ lies in the yz-plane and has slope $\frac{4}{3}$. An orthogonal vector with slope

(b) The unit tangent vector is

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|}.$$

Note that

$$|\mathbf{v}(t)| = \sqrt{9\sin^2 t + 9\cos^2 t + 7} = \sqrt{9 + 7} = 4.$$

Thus

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$$\mathbf{T}(t) = \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|} = \frac{1}{4}\left(\;3\sin t\right)\mathbf{i} + (3\cos t)\,\mathbf{j} + \sqrt{7}\,\mathbf{k}$$

$$= \frac{3}{4}\sin t\;\mathbf{i} + \frac{3}{4}\cos t\;\mathbf{j} + \frac{\sqrt{7}}{4}\mathbf{k}.$$

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(c) The displT2J 0.8.9091 Tf -280Tq1 0 0 1 26r0(displT65 Tf 319.945 557.425 7.9701 Tf 12.105 10.98g63/F35 7.9